EFFECTIVE ENERGY FUNCTIONALS FOR ELASTIC THIN FILMS IN THE SETTING OF ORLICZ–SOBOLEV SPACES

WŁODZIMIERZ LASKOWSKI
WEST POMERANIAN UNIVERSITY OF TECHNOLOGY, SZCZECIN (POLAND)

We consider an elastic membrane - thin film as a bounded open subset ω of $\mathbb{R}^2$ with Lipschitz boundary. The set $\Omega_\varepsilon := \omega \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2}) \subset \mathbb{R}^3$ for a small thickness ε is considered as an elastic cylinder approximate to the membrane ω. A three-dimensional deformation $U : \Omega_\varepsilon \to \mathbb{R}^3$ defined on the thin cylinder Ω_ε has the re-scaled elastic total energy represented by the difference of the re-scaled bulk and surface energies

$$E_\varepsilon(U) = \frac{1}{\varepsilon^\alpha} \int_{\Omega_\varepsilon} W(DU)dx - \frac{1}{\varepsilon^\beta} Q_\varepsilon(U)$$

for some $\alpha, \beta \geq 0$, where $W : \mathbb{R}^{3\times 3} \to \mathbb{R}$ is so-called the energy density function satisfying the growth and coercivity conditions

$$\frac{1}{C}(M(\|F\|) - 1) \leq W(F) \leq C(1 + M(\|F\|)) \quad (\forall F \in \mathbb{R}^{3\times 3})$$

for some $C > 0$. Here $M\colon \mathbb{R} \to [0, \infty)$ is some non-power Orlicz-Young convex function.

We present the effective energy functional for the thin film ω obtained, by Γ-convergence and $3D$-$2D$ dimension reduction techniques applied to the sequence of the re-scaled total energy integral functionals of the elastic cylinders Ω_ε as the thickness ε goes to 0. The existence of minimizers of the energy functional for the membrane ω is established by showing that some sequence of re-scaled minimizers weakly converges in an appropriate Orlicz-Sobolev space to a minimizer of the membrane energy functional.

Our results [1] extend classical results established by H. Le Dret and A. Raoult in [2] for the case of membranes in the reflexive Sobolev space setting with $M(t) = |t|^p$ for some $p \in (1, \infty)$. We would also like to mention about our new results concerning effective energy functionals for elastic thin films with two and three dimensional bending moment.

References

[1] W. Laskowski and H. T. Nguyen, *Effective energy integral functionals for thin films in the Orlicz-Sobolev space setting*, Demonstratio Math. 46 No. 3 (2013), 589-608.

[2] H. Le Dret and A. Raoult, *The nonlinear membrane model as variational limit of nonlinear threedimensional elasticity*, J. Math. Pures Appl. 74 (1995), 549-578.