Paley spaces.

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Let $\sum_{n=1}^{\infty} c_n(x)h_n$ be the Fourier-Haar expansion of an arbitrary function $x \in L_1[0,1]$. Then its Paley (square) function is defined as follows:

$$Px(t) := \Big(\sum_{n=1}^{\infty} (c_n(x)h_n(t))^2 \Big)^{1/2}, \quad 0 \le t \le 1.$$

Given an rearrangement invariant function space X on $[0,1]$ denote by $P(X)$ the corresponding Paley space consisting of all functions $x \in L_1[0,1]$ such that $Px \in X$. Let us equip $P(X)$ with the natural norm

$$\|x\|_{P(X)} := \|Px\|_X.$$

In particular, if $X = L_1$ we obtain the well-known dyadic space H^1, predual to the dyadic BMO-space.

We will discuss various properties of the Paley spaces which are caused by some interrelations between properties of Haar functions and rearrangement invariant spaces. Along with other results, we present necessary and sufficient conditions for the spaces $P(X)$ to have the lattice property (both from isometric and isomorphic points of view), give a reflexivity criterion for them. It turns out that Rademacher functions span a 1-complemented subspace in $P(X)$ for any rearrangement invariant space X.

This is a joint work with E. Semenov.