FUNCTION SPACES XI

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Invariant means in the theory of functional equations and inequalities

A b s t r a c t

Banach limits of bounded real sequences should be viewed as an archetype of the notion of invariant means whose theory belongs nowadays to the standard tools of abstract harmonic analysis. On the other hand invariant means yield also a substitute of the notion of an integral being invariant with respect to the shifts of arguments of the function that is integrated. Although the classical notion of an invariant mean is used to be defined as a special real linear functional (J. von Neumann, M. M. Day), application considerations (in particular in the theory of functional equations and inequalities) forced researchers to seek for various generalizations of this notion to the case of vector valued maps with values in semireflexive locally convex spaces as well as in normed linear spaces with Hahn-Banach extension property or in boundedly complete linear lattices.The key issue is here the question of the existence of a suitably defined invariant mean.

Assuming that we are given a classical scalar valued mean m on the space $\mathcal{B}(S, \mathbb{R})$ of bounded real functions defined on a semigroup $(S, +)$ and a locally convex Hausdorff linear topological space X, by a mean $M(f)$ of a function $f : S \longrightarrow X$ we shall understand an element $M(f)$ of the space X, such that

$$x^*(M(f)) = m(x^* \circ f)\,, \quad x^* \in X^*\,.$$

The uniqueness of $M(f)$ results from the fact that the dual space X^* separates points. One of possible sufficient conditions for its existence is the requirement that $\operatorname{cl}\operatorname{conv}(f(S))$ is weakly compact.

Among others, we shall present applications of that type results while looking for additive selections of some multivalued maps and stability problems for group homomorphisms.