Factorization of some Banach function spaces

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The well-known factorization theorem of Lozanovskiĭ may be written in the form $L^1 \equiv E \odot E'$, where $\odot$ means the pointwise product of Banach ideal spaces and E' is a Köthe dual of E. A natural generalization of this problem would be the question when one can factorize F through E, i.e., when

$$F \equiv E \odot M(E, F)\,, \tag{1}$$

where $M(E, F)$ is the space of pointwise multipliers from E to F. Given two Banach ideal spaces E and F on a measure space (Ω, Σ, μ) define the pointwise product space $E \odot F$ as

$$E \odot F = \{x \cdot y : x \in E \text{ and } y \in F\}\,.$$

with a functional $\|\cdot\|_{E\odot F}$ defined by the formula

$$\|z\|_{E\odot F} = \inf\{\|x\|_E\,\|y\|_F : z = xy, x \in E, y \in F\}\,. \tag{2}$$

Moreover, the space of multipliers $M\,(E, F)$ is defined as

$$M\,(E, F) = \left\{x \in L^0 : xy \in F \text{ for each } y \in E\right\}$$

with the operator norm

$$\|x\|_{M(E,F)} = \sup_{\|y\|_E = 1} \|xy\|_F\,.$$

We will discuss some basic properties of the constructions $M(E, F)$ and $E \odot F$. It also seems to be useful to have equality (1) with just the equivalence of norms, that is, $F = E \odot M(E, F)$. We present results concerning such factorization in selected classes of Banach function spaces. This can be done by finding $M(E, F)$ and $E \odot F$ separately. Thus the form of spaces $M(E, F)$ and $E \odot F$ for concrete class of Banach spaces may be applied at this point.

The talk is based on the papers [1] and [2].

References

[1] P. Kolwicz, K. Leśnik and L. Maligranda, Pointwise multipliers of Calderón-Lozanovskiĭ spaces, Math. Nachr. 286 (8-9) (2013), 876–907.

[2] P. Kolwicz, K. Leśnik and L. Maligranda, Pointwise products of some Banach function spaces and factorization, J. Funct. Anal. 266 (2) (2014) 616-659.