

The Carleson maximal operator and convergence of Fourier series

Mieczysław Mastyło

Adam Mickiewicz University in Poznań; and Institute of Mathematics,
Polish Academy of Sciences (Poznań branch)

Abstract. We provide a new approach which allows us to prove the novel interpolation estimates for the Carleson maximal operators generated by the partial sums of the multiple Fourier series and all its conjugate series over cubes defined on the d -dimensional torus \mathbb{T}^d . Combing these estimates we show that these operators are bounded from a variant of the Arias-de-Reyna space QA^d to the weak L^1 -space on \mathbb{T}^d . This implies that the multiple Fourier series of every function $f \in QA^d$ and all its conjugate series converge over cubes almost everywhere. By a close analysis of the space QA^d we prove that it contains a Lorentz space that strictly contains the Orlicz space $L(\log L)^d \log \log \log L(\mathbb{T}^d)$. As a consequence we obtain an improvement of a deep theorem proved by Antonov which was the best known result on the convergence of multiple Fourier series over cubes. The talk is based on the joint work with L. Rodríguez-Piazza.