

Directional convexity and characterizations of Beta and Gamma functions

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ABSTRACT. The logarithmic convexity of restrictions of the Beta functions to rays parallel to the main diagonal and the functional equation

$$\phi(x+1) = \frac{x(x+k)}{(2x+k+1)(2x+k)}\phi(x), \quad x > 0,$$

for $k > 0$ allow to get a characterizations of the Beta function. This fact and a notion of the beta-type function lead to a new characterization of the Gamma function.

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