On local geometric structure of symmetric spaces.

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Let L^0 be a set of all (equivalence classes of) extended real valued m-measurable functions on $I = [0, \alpha)$, where $0 < \alpha \leq \infty$. For any $x \in L^0$ we denote $x^*(t) = \inf\{\lambda > 0 : m(|x| > \lambda) \leq t\}$, $x^{**}(t) = \frac{1}{t}\int_0^t x^*(s)ds$ for $t > 0$. Let $1 \leq p < \infty$ and $w \geq 0$ be a weight function. The Lorentz space $\Gamma_{p,w}$ is subspace of all measurable functions L^0 such that $\|x\|_{\Gamma_{p,w}} := \left(\int_0^\alpha x^{**p}(t)w(t)dt\right)^{1/p} < \infty$. For more details about the properties of $\Gamma_{p,w}$ we refer to [2, 3, 4, 5, 6].

We present several results in symmetric spaces devoted to local approach of strict K-monotonicity, strict monotonicity and the Kadec-Klee property for global convergence in measure. We also characterize the complete criteria for a point of upper K-monotonicity in Lorentz spaces $\Gamma_{p,w}$ with the degenerated weight function w.

Next we answer the essential question whether a point of local uniform rotundity (shortly an LUR point) can be equivalently considered only on the positive cone E^+ of a Banach function space E. We also investigate under which conditions a point x in a symmetric space E is an LUR point if and only if its decreasing rearrangement x^* is an LUR point.

We characterize a complete criteria of local uniform rotundity in Lorentz spaces $\Gamma_{p,w}$. We present the class of Banach spaces constructed by the K-method of interpolation which possess LUR property. Finally, we discuss the necessary conditions for a point of local uniform rotundity in Lorentz spaces $\Gamma_{p,w}$ with the degenerated weight function w. Presented results come from the paper [1].

References

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