Generalization of fractional integrals in central Morrey spaces

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For $0 < \alpha < n$ and $f \in L^1_{\rm loc}(\mathbb{R}^n)$, let I_α be a fractional integral, i.e.,

$$I_\alpha f(x) = \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}}\, dy$$

and $\tilde{I}_\alpha$ be a modified fractional integral, i.e.,

$$\tilde{I}_\alpha\, f(x) = \int_{\mathbb{R}^n} f(y) \left(\frac{1}{|x-y|^{n-\alpha}} - \frac{1-\chi_{Q_1}(y)}{|y|^{n-\alpha}} \right) dy,$$

where χ_{Q_1} is the characteristic function of Q_1, and for $1 \le p < \infty$ and $-n/p \le \lambda < \infty$, let $B^{p,\lambda}(\mathbb{R}^n)$ be a (non-homogeneous) central Morrey space, i.e.,

$$B^{p,\lambda}(\mathbb{R}^n) = \{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{B^{p,\lambda}} < \infty \},$$

where

$$\|f\|_{B^{p,\lambda}} = \sup_{r \ge 1} \frac{1}{r^\lambda} \left(\frac{1}{|Q_r|} \int_{Q_r} |f(y)|^p\, dy \right)^{1/p}.$$

Then, for $0 < \alpha < n$ and $1 \le p < \infty$, the following are known:

- when $-n/p \le \lambda < -\alpha$,
  I_α is well-defined and bounded for $B^{p,\lambda}(\mathbb{R}^n)$;
- when $-n/p \le \lambda < 1-\alpha$,
  $\tilde{I}_\alpha$ is well-defined and bounded for $B^{p,\lambda}(\mathbb{R}^n)$.

In this talk, for the whole of λ such that $-n/p \le \lambda < \infty$, we will extend the results of boundedness of I_α for $B^{p,\lambda}(\mathbb{R}^n)$. In order to do this, we will

- introduce the "new" function spaces;
- define the "new" modification of I_α which is well-defined for $B^{p,\lambda}(\mathbb{R}^n)$, when $\lambda \ge 1-\alpha$.

Note: For $r > 0$, $Q_r = \{y \in \mathbb{R}^n : |y| < r\}$ or $Q_r = \{y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n : \max_{1 \le i \le n} |y_i| < r\}$.