Uniform convergence of double sine series

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The fundamental theorem in the theory of the uniform convergence of sine series is due to Chaundy and Jolliffe from 1916 (see [1]).

Theorem. *Suppose that $b_n \geq b_{n+1}$ and $b_n \to 0$. Then a necessary and sufficient condition for the uniform convergence of the series*

$$\sum_{n=1}^{\infty} b_n \sin nk$$

is $nb_n \to 0$.

Several authors gave conditions for this problem supposing that coefficients are monotone, non-negative or more recently, general monotone (see [2] and [3], for example). There are also result for the regular convergence of double sine series to by uniform in case the coefficients are monotone or general monotone double sequences. In this presentations we give new sufficient conditions for the uniformity of the regular convergence of double sine series, which are necessary as well in case the coefficients are non-negative.

The new results also bring necessary and sufficient conditions for the uniform regular convergence of double trigonometric series in complex form. We shall generalize those results defining a new class of double sequences for the coefficients.

References

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