SELECTED TOPICS ON THE WEAK TOPOLOGY OF BANACH SPACES

JERZY KĄKOL

Abstract. Corson (1961) started a systematic study of certain topological properties of the weak topology w of Banach spaces E. This line of research provided more general classes such as reflexive Banach spaces, Weakly Compactly Generated Banach spaces and the class of weakly K-analytic and weakly K-countably determined Banach spaces. On the other hand, various topological properties generalizing metrizability have been studied intensively by topologists and analysts. Let us mention, for example, the first countability, Fréchet–Urysohn property, sequentiality, k-space property, and countable tightness. Each property (apart the countable tightness) forces a Banach space E to be finite-dimensional, whenever E_w is assumed to be a space of the above type. This is a simple consequence of a theorem of Schlüchtermann and Wheeler that an infinite-dimensional Banach space is never a k-space in the weak topology. These results show also that the question when a Banach space endowed with the weak topology is homeomorphic to a certain fixed model space from the infinite-dimensional topology is very restrictive and motivated specialists to detect the aforementioned properties only for some natural classes of subsets of E, e.g., balls or bounded subsets of E. We collect some classical and recent results of this type, and characterize those Banach spaces E whose unit ball B_w is $k_{\mathbb{R}}$-space or even has the Ascoli property. Some basic concepts from probability theory and measure-theoretic properties of the space ℓ_1 will be used. Another line of research was essentially related with the concept of networks and k-networks. We present some classical, as well as quite new results characterizing those Banach spaces E for which E_w with the weak topology admits a countable k-network or even a σ-locally finite k-network. A full characterization for this effect will be presented for Fréchet spaces $C_c(X)$ with the compact-open topology. Possible applications to the renorming theory will be mentioned.

University of Poznan, Poland
E-mail address: kakol@amu.edu.pl