Heat and Navier-Stokes equations in supercritical function spaces

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We deal with the Navier-Stokes equations in the version of

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{u} - \Delta_x \mathbf{u} + \mathbb{P}\,\mathrm{div}(\mathbf{u}\otimes\mathbf{u}) &= 0 \qquad \text{in } \mathbb{R}^n \times (0,T), \\ \mathbf{u}(\cdot,0) &= \mathbf{u}_0 \qquad \text{in } \mathbb{R}^n, \end{aligned}$$

and their reduction to the scalar nonlinear heat equation

$$\begin{aligned} \frac{\partial}{\partial t}u - \Delta_x u - Du^2 &= 0, \qquad \text{in } \mathbb{R}^n \times (0,T), \\ u(\cdot,0) &= u_0, \qquad \text{in } \mathbb{R}^n, \end{aligned}$$

where $0 < T \leq \infty$. Here $\mathbb{P}$ is the Leray projector based on the Riesz transform.

Starting with the heat equation we assume for the initial data $u_0 \in A^\sigma_{p,q}(\mathbb{R}^n)$ with $A \in \{B, F\}$, that $\sigma - \frac{n}{p} > -1$, $1 \leq p,q \leq \infty$ and additional requirements on q if $A = B$. These spaces cover all supercritical cases for the initial data in the context of Navier-Stokes equations. We ask for solutions u belonging to some spaces $L_{2v}((0,T),\, \frac{a}{2},\, A^s_{p,q}(\mathbb{R}^n)) \cap C([0,T), A^\sigma_{p,q}(\mathbb{R}^n))$, that is for so called strong solutions with

$$\|u|L_{2v}((0,T),\, \frac{a}{2},\, A^s_{p,q}(\mathbb{R}^n))\| = \left(\int_0^T t^{av} \|u(\cdot,t)|A^s_{p,q}(\mathbb{R}^n)\|^{2v} \mathrm{d}t \right)^{1/2v} < \infty$$

where $1 \leq p,q \leq \infty$ and $-1 + n/p < s < n/p$.

Finally we apply the gained results to the Navier-Stokes equations, using the heat equation as its scalar model case obtained by means of the Leray projector.

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