THE DAUGAVET PROPERTY AND RELATED PROPERTIES IN THE MUSIELAK-ORLICZ FUNCTION SPACES

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Abstract. Let $(X, \|\cdot\|)$ be a Banach space and $T : X \to X$ be a bounded linear operator. The equation

$$\|T + I\| = 1 + \|T\|, \tag{DE}$$

where I is the identity operator from X to X, is called the Daugavet equation. If the Daugavet equation is satisfied by all rank one operators then X is said to have the Daugavet property. It is known that if X has the Daugavet property then the equation (DE) is satisfied by every weakly compact operator.

It has been showed recently that among all rearrangement invariant function spaces over a non-atomic finite measure only the spaces L_1 and L_∞ have the Daugavet property [Acosta, M. D., Kamińska, A., Mastyło, M. 2013]. Inspired by that result we study the Daugavet property in the class of Musielak-Orlicz function spaces on a σ-finite non-atomic complete measure space, which are not rearrangement invariant in general and contain the variable exponent Lebesgue spaces (Nakano spaces) as well as the Orlicz spaces as proper subclasses. We give a characterization of Musielak-Orlicz spaces (equipped with either the Luxemburg norm or the Orlicz norm) with the Daugavet property.

We also study closely related notions of diameter 2 properties and octahedral norms in these spaces. We say that a Banach space $(X, \|\cdot\|)$ has the *strong diameter* 2 *property* if every convex combination of slices of the unit ball of X has diameter 2. In case of the Orlicz norm, we characterize all the Musielak-Orlicz spaces which have the strong diameter 2 property.

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