Smoothness Morrey Spaces of regular distributions, and their envelopes

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The classical Morrey spaces $\mathcal{M}_{u,p}$, $0 < p \leq u < \infty$, consist of all locally p-integrable functions f such that

$$\|f|\mathcal{M}_{u,p}(\mathbb{R}^n)\| = \sup_{x\in\mathbb{R}^n, R>0} R^{\frac{n}{u}-\frac{n}{p}} \left(\int_{B(x,R)} |f(y)|^p \, \mathrm{d}y\right)^{1/p}$$

is finite, where $B(x,R)$ are the usual balls centered at $x \in \mathbb{R}^n$ with radius $R > 0$. They are part of the wider class of Morrey-Campanato spaces and can be considered as an extension of the scale of L_p spaces. Built upon these basic spaces Besov-Morrey spaces $\mathcal{N}^s_{u,p,q}$ and Triebel-Lizorkin-Morrey spaces $\mathcal{E}^s_{u,p,q}$ attracted some attention in the last years, in particular in connection with Navier-Stokes equations. Closely related to these scales are the spaces of Besov-type $B^{s,\tau}_{p,q}$ and of Triebel-Lizorkin type $F^{s,\tau}_{p,q}$, $\tau \geq 0$, which coincide with their classical counterparts for $\tau = 0$.

We consider such different scales of smoothness spaces of Morrey type and determine their growth and continuity envelopes. In some cases a specific behaviour appears which is different from the 'classical' situation in Besov or Triebel-Lizorkin spaces. Moreover, we are able to give a first (almost complete) characterisation when such spaces consist of regular distributions only.

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