On the minimal property of de la Vallée Poussin's operator

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Abstract

Let $\mathcal{C}_0(2\pi)$ denote the space of all continuous, 2π-periodic functions equipped with the supremum norm. Let Π_n denote the space of all trigonometric polynomials of degree less than or equal to n. The Fourier projection $F_n : \mathcal{C}_0(2\pi) \to \Pi_n$ is defined by

$$F_n(f)(t) = \frac{1}{2\pi} \int_0^{2\pi} f(s) D_n(t-s) ds, \tag{1}$$

where D_n is the Dirichlet kernel

$$D_n(t) = \sum_{k=-n}^{n} e^{ikt}. \tag{2}$$

It is well known by the classical result of Lozinski that Fourier operator F_n has the minimal norm among all projections from $\mathcal{C}_0(2\pi)$ onto Π_n. If we replace $\mathcal{C}_0(2\pi)$ by $L_1[0, 2\pi]$ the Lozinski theorem stays true.
In 1918 de la Vallée Poussin introduced the following operator.

Definition 0.1 *De la Vallée Poussin's operator $H_n : X \to \Pi_{2n-1}$ is given by*

$$H_n(f)(t) = \frac{1}{n} \sum_{k=n}^{2n-1} F_k(f)(t) = \frac{1}{2\pi} \int_0^{2\pi} f(s) V_n(t-s) ds, \tag{3}$$

where

$$V_n(t) = \frac{1}{n} \sum_{k=n}^{2n-1} D_n(t) = \frac{(\sin(nx))^2 - (\sin(\frac{n}{2}x))^2}{n(\sin(\frac{1}{2}x))^2}. \tag{4}$$

In our talk we present a certain generalization of the Lozinski theorem and based on it and a behaviour of the zeros of the kernel V_n we show that de la Vallée Poussin's operator has the minimal norm in the set of generalized projections

$$\mathcal{P}_{\Pi_n}(X, \Pi_{2n-1}) = \{P \in \mathcal{L}(X, \Pi_{2n-1}) : P|_{\Pi_n} \equiv id\}, \tag{5}$$

where $X = \mathcal{C}_0(2\pi)$ or $X = L_1[0, 2\pi]$.

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