s-numbers of compact embeddings of function spaces on quasi-bounded domains.

Sobolev embeddings

$$B^{s_1}_{p_1,q_1}(\mathbb{R}^d) \hookrightarrow B^{s_2}_{p_2,q_2}(\mathbb{R}^d),$$

where $B^s_{p,q}(\mathbb{R}^d)$ are Besov spaces, are never compact. Therefore if we want to investigate asymptotic behaviour of s-numbers of this embeddings than we need to modify the Besov spaces. For this purpose we will use quasi-bounded domains. Domain Ω in $\mathbb{R}^d$ is called quasi-bounded if

$$\lim_{x\in\Omega, |x|\to\infty} d(x, \partial\Omega) = 0.$$

In particular, each bounded domain is quasi-bounded. However we also suppose that Ω is a uniformly E-porous domain because there exists isomorphism between space $\bar{B}^s_{p,q}(\Omega)$ and $\ell_q\big(2^{j(s-d/p)}\ell_p^{M_j}\big)$. Therefore

$$s_k(\bar{B}^{s_1}_{p_1,q_1}(\Omega) \hookrightarrow \bar{B}^{s_2}_{p_2,q_2}(\Omega)) \sim s_k(\ell_{q_1}(2^{j\delta}\ell_{p_1}^{M_j}) \hookrightarrow \ell_{q_2}(\ell_{p_2}^{M_j})),$$

for any s-numbers. The above embedding is compact if

$$s_1 - s_2 - d\bigg(\frac{1}{p_1} - \frac{1}{p_2}\bigg) > b(\Omega)\bigg(\frac{1}{p_2} - \frac{1}{p_1}\bigg)_+,$$

where $b(\Omega)$ is so called box packing constant. So investigation of asymptotic behaviour of s-numbers of Sobolev embeddings on quasi-bounded domains is reduced to the case of investigation of asymptotic behaviour of s-numbers of Sobolev embeddings of above sequence spaces.